RAPID ADIABATIC PREPARATION OF INJECTIVE PEPS AND GIBBS STATES

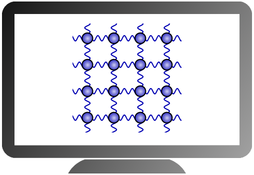
ARXIV:1508.00570 [QUANT-PH]

Yimin Ge, András Molnár, Ignacio Cirac

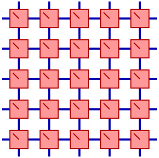
QIP 2016, 15.01.2016

Max Planck Institute of Quantum Optics, Garching, Germany

Quantum simulation

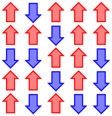


Quantum many-body systems



State preparation algorithm

Classical Gibbs distributions

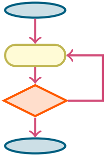


Statistical physics

Optimisation problems

Machine learning

Quantum algorithms



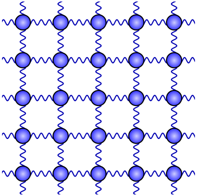
OUTLINE

1. Introduction
    - PEPS
    - Gibbs states
    - Parent Hamiltonians
2. Naive algorithm
    - Adiabatic theorem
    - Hamiltonian simulation
3. Algorithm
    - Improved adiabatic theorem
    - Local changes
    - Lieb-Robinson localisation
4. Runtime & gap assumption

INTRODUCTION

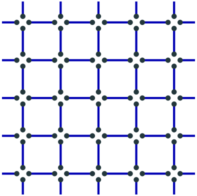
PROJECTED ENTANGLED PAIR STATES - PEPS

- Quantum system with N particles - $\dim \mathcal{H} = d^N$
  - Too large to describe efficiently
- Ground states of local gapped Hamiltonians
  - Special entanglement properties $\rightarrow$ better description?



PROJECTED ENTANGLED PAIR STATES - PEPS

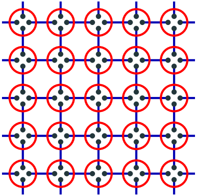
- Quantum system with N particles - $\dim \mathcal{H} = d^N$
  - Too large to describe efficiently
- Ground states of local gapped Hamiltonians
  - Special entanglement properties $\rightarrow$ better description?



 $= \sum_{i=1}^{D} |ii\rangle$

PROJECTED ENTANGLED PAIR STATES - PEPS

- Quantum system with N particles - $\dim \mathcal{H} = d^N$
    - Too large to describe efficiently
- Ground states of local gapped Hamiltonians
    - Special entanglement properties $\quad \rightarrow \quad$ better description?



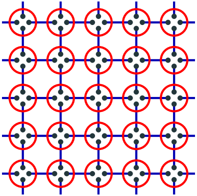
●—● $= \sum_{i=1}^{D} |ii\rangle$

○ $= Q_\lambda : (\mathbb{C}^D)^{\otimes 4} \rightarrow \mathbb{C}^d$

$|\psi_{\text{PEPS}}\rangle = \bigotimes_{\text{sites } \lambda} Q_\lambda \bigotimes_{\text{edges}}$ ●—●

PROJECTED ENTANGLED PAIR STATES - PEPS

- Quantum system with N particles - $\dim \mathcal{H} = d^N$
  - Too large to describe efficiently
- Ground states of local gapped Hamiltonians
  - Special entanglement properties $\quad\rightarrow\quad$ better description?



$$\bullet\!\!-\!\!\bullet \;=\; \sum_{i=1}^{D} |ii\rangle$$

$$\bigcirc \;=\; Q_\lambda : (\mathbb{C}^D)^{\otimes 4} \rightarrow \mathbb{C}^d$$

$$|\psi_{\text{PEPS}}\rangle \;=\; \bigotimes_{\text{sites } \lambda} Q_\lambda \bigotimes_{\text{edges}} \bullet\!\!-\!\!\bullet$$

Injective PEPS

Q_λ invertible $\;\forall\lambda$

(generic case)

Gibbs state

- $H = \sum_\mu h_\mu \qquad [h_\mu, h_\nu] = 0$
- $\rho_\beta = \frac{1}{Z} e^{-\beta H} \qquad Z = \mathrm{Tr}\, e^{-\beta H}$
- $\beta = \text{Temperature}^{-1}$

GIBBS STATES - PURIFICATION

Gibbs state

- $H = \sum_\mu h_\mu \qquad [h_\mu, h_\nu] = 0$
- $\rho_\beta = \frac{1}{Z} e^{-\beta H} \qquad Z = \mathrm{Tr}\, e^{-\beta H}$
- $\beta = \text{Temperature}^{-1}$

Purification

- $|\Psi\rangle = \frac{1}{\sqrt{Z}} \left(e^{-\beta H/2} \otimes \mathbb{1} \right) |\phi\rangle^{\otimes N}$
- $|\phi\rangle = \frac{1}{\sqrt{d}} \sum_i |ii\rangle$
- $\mathrm{Tr}_{\text{Ancilla}} |\Psi\rangle\langle\Psi| = \rho_\beta$

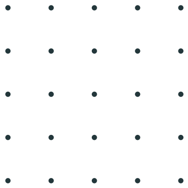
GIBBS STATES - PURIFICATION

Gibbs state

- $H = \sum_\mu h_\mu \qquad [h_\mu, h_\nu] = 0$
- $\rho_\beta = \frac{1}{Z} e^{-\beta H} \qquad Z = \mathrm{Tr}\, e^{-\beta H}$
- $\beta = \text{Temperature}^{-1}$

Purification

- $|\Psi\rangle = \frac{1}{\sqrt{Z}} \left(e^{-\beta H/2} \otimes \mathbb{1}\right) |\phi\rangle^{\otimes N}$
- $|\phi\rangle = \frac{1}{\sqrt{d}} \sum_i |ii\rangle$
- $\mathrm{Tr}_{\text{Ancilla}} |\Psi\rangle\langle\Psi| = \rho_\beta$



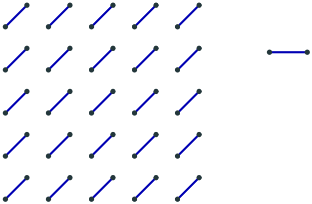
GIBBS STATES - PURIFICATION

Gibbs state

- $H = \sum_\mu h_\mu \qquad [h_\mu, h_\nu] = 0$
- $\rho_\beta = \frac{1}{Z} e^{-\beta H} \qquad Z = \mathrm{Tr}\, e^{-\beta H}$
- $\beta = \text{Temperature}^{-1}$

Purification

- $|\Psi\rangle = \frac{1}{\sqrt{Z}} \left(e^{-\beta H/2} \otimes \mathbb{1} \right) |\phi\rangle^{\otimes N}$
- $|\phi\rangle = \frac{1}{\sqrt{d}} \sum_i |ii\rangle$
- $\mathrm{Tr}_{\text{Ancilla}} |\Psi\rangle\langle\Psi| = \rho_\beta$



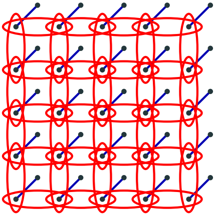
$$= \sum_{i=1}^{d} |ii\rangle$$

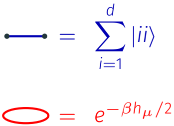
Gibbs state

- $H = \sum_\mu h_\mu \qquad [h_\mu, h_\nu] = 0$
- $\rho_\beta = \frac{1}{Z} e^{-\beta H} \qquad Z = \mathrm{Tr}\, e^{-\beta H}$
- $\beta = \text{Temperature}^{-1}$

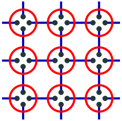
Purification

- $|\Psi\rangle = \frac{1}{\sqrt{Z}} \left(e^{-\beta H/2} \otimes \mathbb{1}\right) |\phi\rangle^{\otimes N}$
- $|\phi\rangle = \frac{1}{\sqrt{d}} \sum_i |ii\rangle$
- $\mathrm{Tr}_{\text{Ancilla}} |\Psi\rangle\langle\Psi| = \rho_\beta$

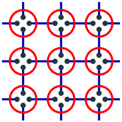




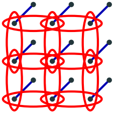
Injective PEPS



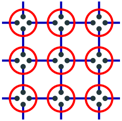
Injective PEPS



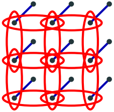
Gibbs state



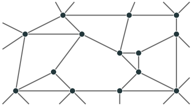
Injective PEPS



Gibbs state

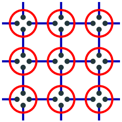


General

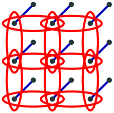


GENERAL CLASS OF STATES – PARENT HAMILTONIANS

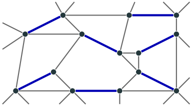
Injective PEPS



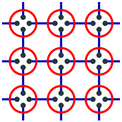
Gibbs state



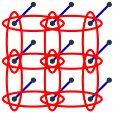
General



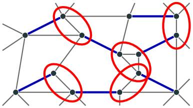
Injective PEPS



Gibbs state

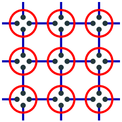


General

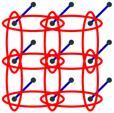


GENERAL CLASS OF STATES - PARENT HAMILTONIANS

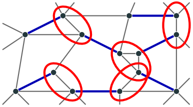
Injective PEPS



Gibbs state

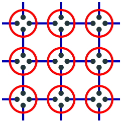


General

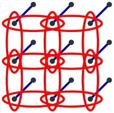


Commuting, finite range, invertible operators Q_λ acting on maximally entangled pairs

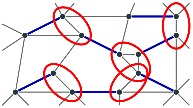
Injective PEPS



Gibbs state

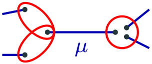


General



Commuting, finite range, invertible operators Q_λ acting on maximally entangled pairs

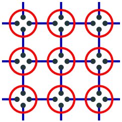
Parent Hamiltonians



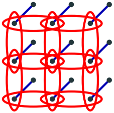
$|\Psi\rangle$

GENERAL CLASS OF STATES – PARENT HAMILTONIANS

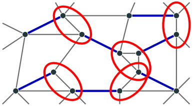
Injective PEPS



Gibbs state

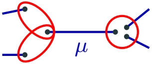


General



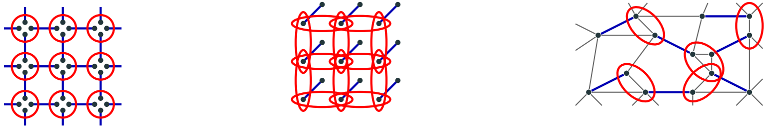
Commuting, finite range, invertible operators Q_λ acting on maximally entangled pairs

Parent Hamiltonians



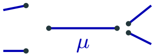
$$\left(\prod_{\lambda : \lambda \cap \mu \neq \emptyset} Q_\lambda^{-1} \right) |\Psi\rangle$$

Injective PEPS | Gibbs state | **General**



Commuting, finite range, invertible operators Q_λ acting on maximally entangled pairs

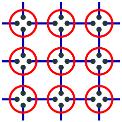
Parent Hamiltonians



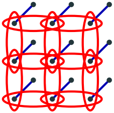
$$\left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right) |\Psi\rangle$$

GENERAL CLASS OF STATES - PARENT HAMILTONIANS

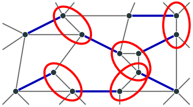
Injective PEPS



Gibbs state

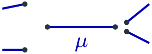


General



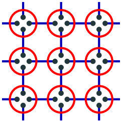
Commuting, finite range, invertible operators Q_λ acting on maximally entangled pairs

Parent Hamiltonians

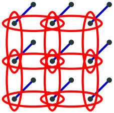


$$P_\mu \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right) |\Psi\rangle$$

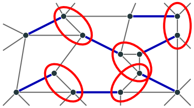
Injective PEPS



Gibbs state

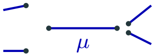


General



Commuting, finite range, invertible operators Q_λ acting on maximally entangled pairs

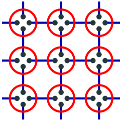
Parent Hamiltonians



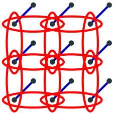
$$P_\mu \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right) |\Psi\rangle = 0$$

GENERAL CLASS OF STATES – PARENT HAMILTONIANS

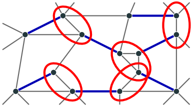
Injective PEPS



Gibbs state

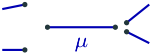


General



Commuting, finite range, invertible operators Q_λ acting on maximally entangled pairs

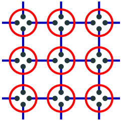
Parent Hamiltonians



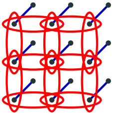
$$\left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1}\right)^\dagger P_\mu \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1}\right)|\Psi\rangle = 0$$

GENERAL CLASS OF STATES - PARENT HAMILTONIANS

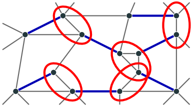
Injective PEPS



Gibbs state



General

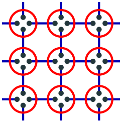


Commuting, finite range, invertible operators Q_λ acting on maximally entangled pairs

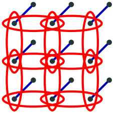
Parent Hamiltonians

$$\sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^\dagger P_\mu \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right) |\Psi\rangle \; = 0$$

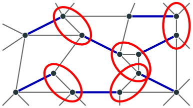
Injective PEPS



Gibbs state



General



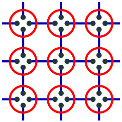
Commuting, finite range, invertible operators Q_λ acting on maximally entangled pairs

Parent Hamiltonians

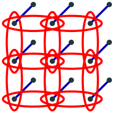
$$G = \sum_{\mu} \Big(\prod_{\lambda : \lambda \cap \mu \neq \emptyset} Q_\lambda^{-1} \Big)^\dagger P_\mu \Big(\prod_{\lambda : \lambda \cap \mu \neq \emptyset} Q_\lambda^{-1} \Big)$$

GENERAL CLASS OF STATES – PARENT HAMILTONIANS

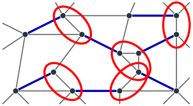
Injective PEPS



Gibbs state



General



Commuting, finite range, invertible operators Q_λ acting on maximally entangled pairs

Parent Hamiltonians

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)^\dagger P_\mu \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)$$

$|\Psi\rangle$ is **unique** ground state of G

A NAIVE ALGORITHM

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda : \lambda \cap \mu \neq \emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda : \lambda \cap \mu \neq \emptyset} Q_{\lambda}^{-1} \Big)$$

A NAIVE ALGORITHM

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

Adiabatic path

$$G(s) = \sum_{\mu} G_{\mu}(s), \quad s \in [0,1]$$

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

Adiabatic path

$$G(s) = \sum_{\mu} G_{\mu}(s), \quad s \in [0,1]$$

Adiabatic theorem

e.g. [Jansen,Ruskai,Seiler '07]

Hamiltonian path $G(s)$ - ground state $|\phi(s)\rangle$ $s \in [0,1]$

Initial state $|\psi(0)\rangle = |\phi(0)\rangle$ $\boldsymbol{\Delta} = \min_s \mathrm{Gap}\, G(s)$

$$i \frac{\mathrm{d}}{\mathrm{d}t} |\psi(t)\rangle = G\left(\frac{t}{\tau}\right) |\psi(t)\rangle \qquad t \in [0,\tau]$$

$\tau = O\left(\frac{\|\dot{G}\|^2}{\varepsilon \boldsymbol{\Delta}^3}\right)$ sufficient for final error $\|\psi(\tau) - \phi(1)\| < \varepsilon$

Parent Hamiltonian

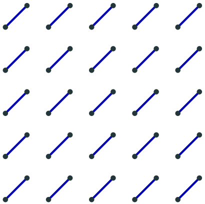
$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

Adiabatic path

$$G(s) = \sum_{\mu} G_{\mu}(s), \quad s \in [0, 1]$$

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

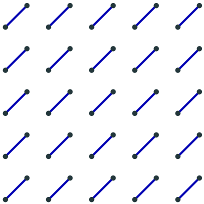


Adiabatic path

$$G(s) = \sum_{\mu} G_{\mu}(s), \quad s \in [0,1]$$

Parent Hamiltonian

$$G = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)$$



$$G(0) = \sum P_{\mu}$$

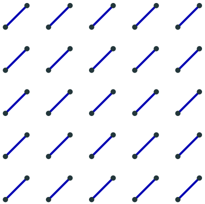
Adiabatic path

$$G(s) = \sum_{\mu} G_{\mu}(s), \quad s \in [0,1]$$

A NAIVE ALGORITHM

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda : \lambda \cap \mu \neq \emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda : \lambda \cap \mu \neq \emptyset} Q_{\lambda}^{-1} \Big)$$



$G(0) = \sum P_{\mu}$

$G(1) = G$

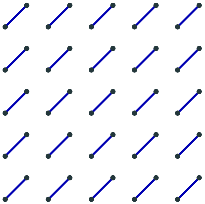
Adiabatic path

$$G(s) = \sum_{\mu} G_{\mu}(s), \quad s \in [0, 1]$$

A NAIVE ALGORITHM

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda : \lambda \cap \mu \neq \emptyset} Q_\lambda^{-1} \Big)^\dagger P_\mu \Big(\prod_{\lambda : \lambda \cap \mu \neq \emptyset} Q_\lambda^{-1} \Big)$$



$$G(0) = \sum P_\mu$$

$$G(1) = G$$

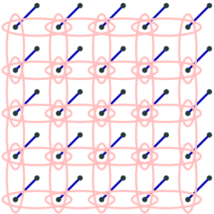
Adiabatic path

$$G(s) = \sum_\mu G_\mu(s), \quad s \in [0, 1]$$

A NAIVE ALGORITHM

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)^\dagger P_\mu \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)$$



$G(0) = \sum P_\mu$

↓

$G(1) = G$

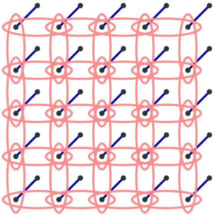
Adiabatic path

$$G(s) = \sum_\mu G_\mu(s), \quad s \in [0,1]$$

A NAIVE ALGORITHM

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)^\dagger P_\mu \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)$$



$G(0) = \sum P_\mu$

↓

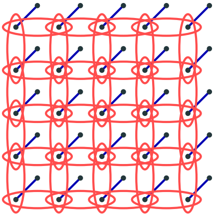
$G(1) = G$

Adiabatic path

$$G(s) = \sum_\mu G_\mu(s), \quad s \in [0,1]$$

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$



Adiabatic path

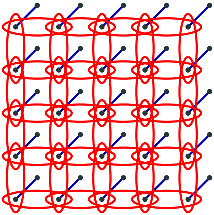
$$G(s) = \sum_{\mu} G_{\mu}(s), \quad s \in [0,1]$$

$$G(0) = \sum P_{\mu}$$

$$G(1) = G$$

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$



$G(0) = \sum P_{\mu}$

$G(1) = G$

Adiabatic path

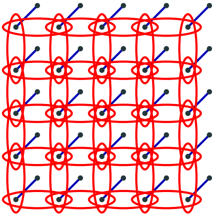
$$G(s) = \sum_{\mu} G_{\mu}(s), \quad s \in [0,1]$$

Parent Hamiltonian

$$G = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)$$

Adiabatic path

$$G(s) = \sum_{\mu} G_{\mu}(s), \quad s \in [0,1]$$



Runtime

$\Delta = \min_s \operatorname{Gap} G(s)$

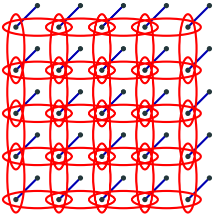
Adiabatic runtime $\quad \tau = O\left(\frac{\|\dot{G}\|^2}{\varepsilon \Delta^3}\right)$

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

Adiabatic path

$$G(s) = \textstyle\sum_{\mu} G_{\mu}(s), \quad s \in [0,1]$$



Runtime

$\Delta = \min_s \text{Gap}\, G(s)$

Adiabatic runtime $\tau = O\left(\frac{N^2}{\varepsilon \Delta^3}\right)$

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

Adiabatic path

$$G(s) = \textstyle\sum_{\mu} G_{\mu}(s), \quad s \in [0,1]$$

(s)

Hamiltonian simulation

e.g. [Berry, et al '14]

Given: Hamiltonian $G(t) \quad t \in [0, \tau]$

Want: Unitary evolution of $G \;\rightarrow\;$ quantum circuit

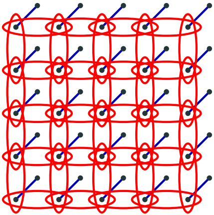
$$\Rightarrow \quad \#\,\text{gates} = \tilde{O}\left((\#\,\text{qubits}) \times \|G\| \times \tau\right)$$

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)^\dagger P_\mu \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)$$

Adiabatic path

$$G(s) = \textstyle\sum_\mu G_\mu(s), \quad s \in [0,1]$$



Runtime

$\boldsymbol{\Delta} = \min_s \text{Gap}\, G(s)$

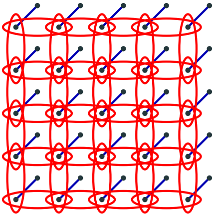
Adiabatic runtime $\tau = O\left(\frac{N^2}{\varepsilon \boldsymbol{\Delta}^3}\right)$

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

Adiabatic path

$$G(s) = \sum_{\mu} G_{\mu}(s), \quad s \in [0,1]$$



Runtime

$\Delta = \min_s \text{Gap}\, G(s)$

Adiabatic runtime $\tau = O\left(\frac{N^2}{\varepsilon \Delta^3}\right)$

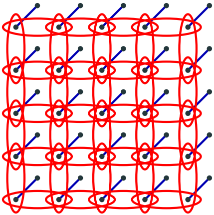
Runtime (# gates) $T = \tilde{O}\left(\tau N^2\right)$

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)^\dagger P_\mu \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)$$

Adiabatic path

$$G(s) = \sum_\mu G_\mu(s), \quad s \in [0,1]$$



Runtime

$\Delta = \min_s \text{Gap}\, G(s)$

Adiabatic runtime $\tau = O\left(\frac{N^2}{\varepsilon \Delta^3}\right)$

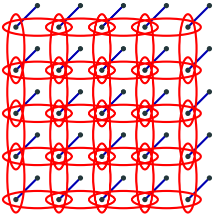
Runtime (# gates) $T = \tilde{O}\left(N^4 \Delta^{-3} \varepsilon^{-1}\right)$

Parent Hamiltonian

$$G = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

Adiabatic path

$$G(s) = \sum_{\mu} G_{\mu}(s), \quad s \in [0,1]$$



Runtime

$\Delta = \min_s \mathrm{Gap}\, G(s)$

Adiabatic runtime $\tau = O\left(\frac{N^2}{\varepsilon \Delta^3}\right)$

Runtime (# gates) $T = \tilde{O}\left(N^4 \Delta^{-3} \varepsilon^{-1}\right)$

Worse than classical algorithms
for classical Gibbs states

IMPROVED ALGORITHM

Summary of results

Algorithm with

Runtime (# gates)	$T = O(N \operatorname{polylog}(N/\varepsilon))$	$\leftarrow$ almost optimal
Circuit depth	$D = O(\operatorname{polylog}(N/\varepsilon))$	

assuming a *uniform gap*

Ingredients

1. Adiabatic theorem with (almost) exponentially small error
2. Sequence of local changes
3. Lieb-Robinson localisation

IMPROVED ALGORITHM

Summary of results

Algorithm with

Runtime (# gates)	$T = O(N\,\mathrm{polylog}(N/\varepsilon))$	$\leftarrow$ almost optimal
Circuit depth	$D = O(\mathrm{polylog}(N/\varepsilon))$	

assuming a *uniform gap*

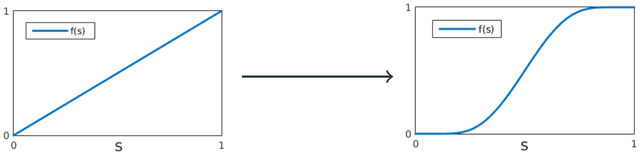
Ingredients

1. Adiabatic theorem with (almost) exponentially small error
2. Sequence of local changes
3. Lieb-Robinson localisation

Main message: **Speedup by *locality***

ADIABATIC THEOREM WITH ALMOST EXPONENTIALLY SMALL ERROR

Smooth reparameterisation $G(s) \quad \to \quad G(f(s))$



Smooth reparameterisation $G(s) \rightarrow G(f(s))$

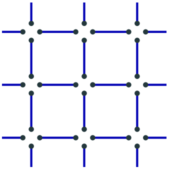
Theorem c.f. [Nenciu '93], [Hagedorn, Joye '02]

- $G(s)$ smooth & all derivatives of $G(s)$ vanish at $s = 0, 1$
- $G(s)$ is in Gevrey class $1 + \alpha$
- $K = |\operatorname{supp} \dot{G}|$

$$\Rightarrow \quad \tau = O\left(\log^{1+\alpha}\left(\frac{K}{\varepsilon\Delta}\right)\frac{K^2}{\Delta^3}\right) \quad \text{sufficient for final error } \varepsilon$$

SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

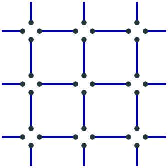


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$$\downarrow$$

$$G_{\text{final}} = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

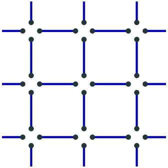


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)$$

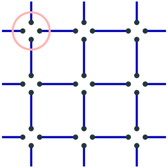


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

↓ Change Q_{λ}'s individually

$$G_{\text{final}} = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

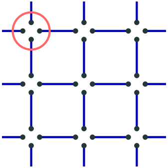


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

Change Q_{λ}'s individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)$$

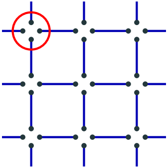


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)$$

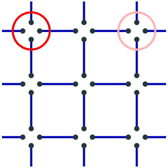


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_λ's individually

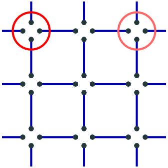
$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^\dagger P_\mu \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)$$



$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

↓ Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

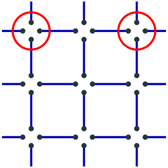


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

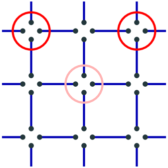
Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)^{\dagger} P_\mu \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)$$



SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$



$\downarrow$ Change Q_λ's individually

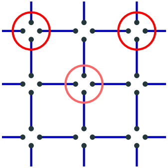
$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda : \lambda \cap \mu \neq \emptyset} Q_\lambda^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda : \lambda \cap \mu \neq \emptyset} Q_\lambda^{-1} \right)$$

SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)$$

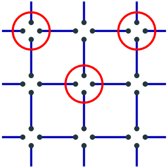


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^{\dagger} P_\mu \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)$$

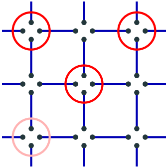


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_{λ}'s individually

$$G_{\text{final}} = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

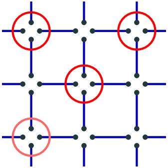


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)$$

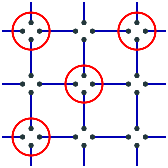


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^{\dagger} P_\mu \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)$$

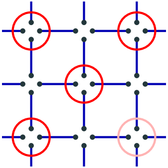


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)$$

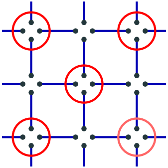


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)$$

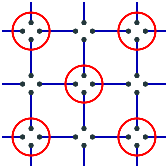


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \Bigg(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Bigg)^\dagger P_\mu \Bigg(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Bigg)$$

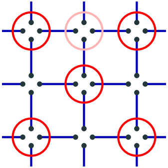


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)$$

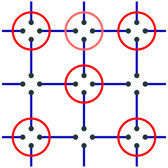


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

Change Q_{λ}'s individually

$$G_{\text{final}} = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Big)$$

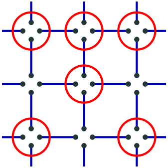


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

Change Q_{λ}'s individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)$$

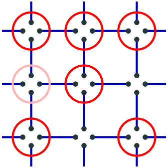


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)$$

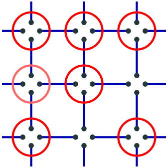


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

↓ Change Q_λ's individually

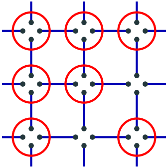
$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)$$



$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_{λ}'s individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)$$

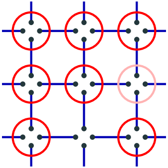


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

Change Q_{λ}'s individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)$$

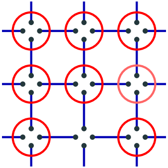


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

↓ Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)$$

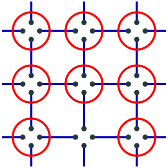


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

↓ Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)$$

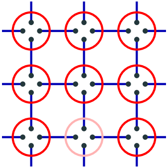


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_{λ}'s individually

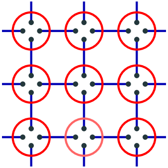
$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \right)$$



$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)$$

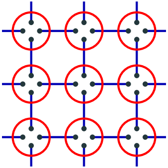


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \Bigg(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Bigg)^{\dagger} P_{\mu} \Bigg(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_{\lambda}^{-1} \Bigg)$$

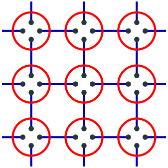


SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^{\dagger} P_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)$$



Δ = minimum gap

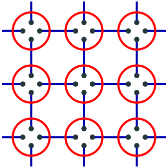
Runtime

Adiabatic runtime $\quad N$ paths $\times O\left(\log^{1+\alpha}(N/\varepsilon\Delta)\Delta^{-3}\right)$

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

$\downarrow$ Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^\dagger P_\mu \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)$$



Δ = minimum gap

Runtime

Adiabatic runtime N paths $\times O\left(\log^{1+\alpha}(N/\varepsilon\Delta)\Delta^{-3}\right)$

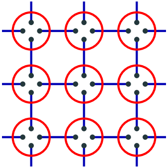
Runtime (# gates) $T = O\left(N^3 \operatorname{polylog}(N/\varepsilon\Delta)\Delta^{-3}\right)$

SEQUENCE OF LOCAL CHANGES

$$G_{\text{initial}} = \sum_{\mu} P_\mu$$

Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)^\dagger P_\mu \left(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \right)$$



$\Delta =$ minimum gap

Runtime

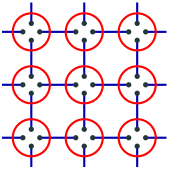
Adiabatic runtime $\quad N$ paths $\times O\left(\log^{1+\alpha}(N/\varepsilon\Delta)\Delta^{-3}\right)$

Runtime (# gates) $\quad T = O\left(N^3 \,\text{polylog}(N/\varepsilon\Delta)\Delta^{-3}\right)$

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)^{\dagger} P_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)$$



Δ = minimum gap

Runtime

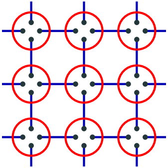
Adiabatic runtime	N paths $\times O\left(\log^{1+\alpha}(N/\varepsilon\Delta)\Delta^{-3}\right)$
Runtime (# gates)	$T = O\left(N^3 \text{ polylog}(N/\varepsilon\Delta)\Delta^{-3}\right)$

Problem: Hamiltonians act on entire system!

$$G_{\text{initial}} = \sum_{\mu} P_{\mu}$$

Change Q_λ's individually

$$G_{\text{final}} = \sum_{\mu} \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)^{\dagger} P_\mu \Big(\prod_{\lambda:\lambda\cap\mu\neq\emptyset} Q_\lambda^{-1} \Big)$$



Δ = minimum gap

Runtime

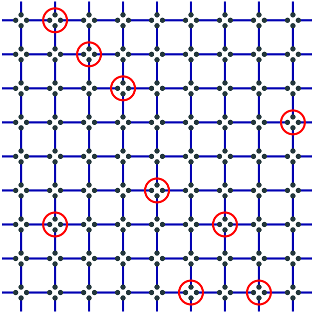
Adiabatic runtime N paths $\times O\left(\log^{1+\alpha}(N/\varepsilon\Delta)\Delta^{-3}\right)$

Runtime (# gates) $T = O\left(N^3 \,\text{polylog}(N/\varepsilon\Delta)\Delta^{-3}\right)$

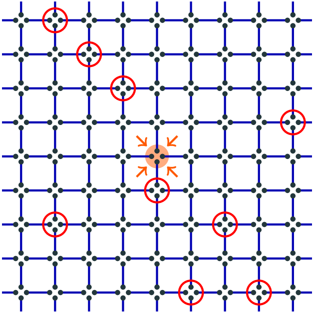
Problem: Hamiltonians act on entire system!

Assume from now: $\Delta = \Omega(1)$

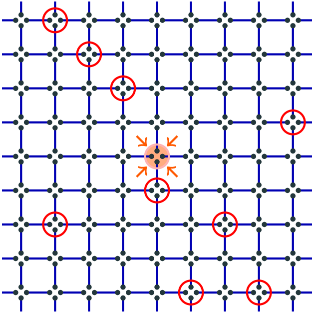
LIEB-ROBINSON LOCALISATION

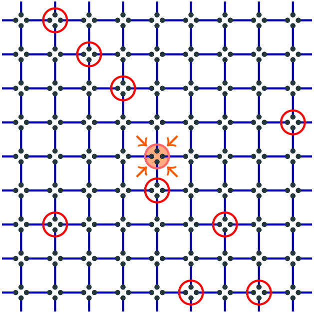


LIEB-ROBINSON LOCALISATION

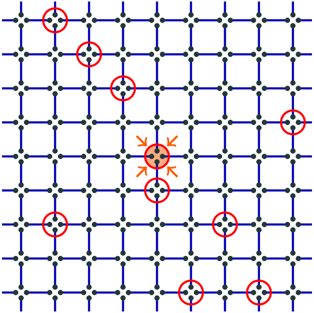


LIEB-ROBINSON LOCALISATION

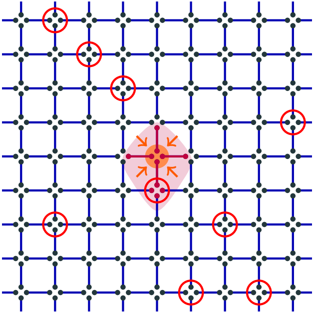




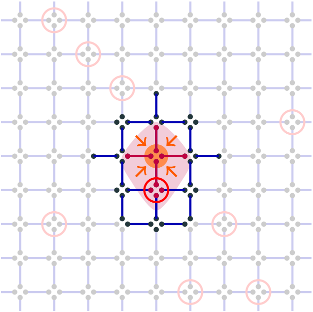
LIEB-ROBINSON LOCALISATION



LIEB-ROBINSON LOCALISATION

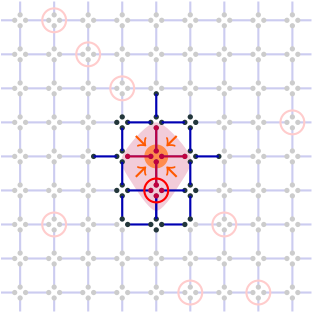


$\text{supp}\, \dot{G}$



Idea

Localise Hamiltonian around $\operatorname{supp} \dot{G}$



Idea

Localise Hamiltonian around $\operatorname{supp} \dot{G}$

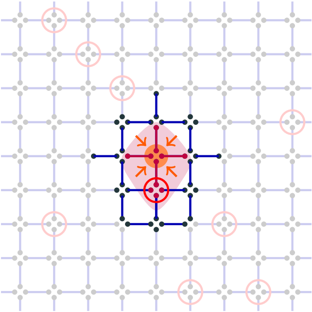
Tool

Lieb-Robinson bound

Assumptions

$\tau = O(\mathrm{polylog}(N/\varepsilon))$

G frustration-free



Idea

Localise Hamiltonian around $\operatorname{supp}\dot{G}$

Tool

Lieb-Robinson bound

Assumptions

$\tau = O(\mathrm{polylog}(N/\varepsilon))$

G frustration-free

Theorem

Hamiltonian terms supported at distance $\gtrsim \mathrm{polylog}\left(\frac{N}{\varepsilon}\right)$ away from $\operatorname{supp}\dot{G}$ have negligible effect on adiabatic evolution

Truncate $\rightarrow$ Hamiltonians act only on $O\left(\mathrm{polylog}\left(\frac{N}{\varepsilon}\right)\right)$ sites

RUNTIME & GAP ASSUMPTION

Runtime

Adiabatic runtime: N paths $\times O\left(\log^{1+\alpha}\left(\frac{N}{\varepsilon}\right)\right)$

Each path: supported on $O\left(\text{polylog}\left(\frac{N}{\varepsilon}\right)\right)$ sites

Runtime

Adiabatic runtime: N paths $\times O\left(\log^{1+\alpha}\left(\frac{N}{\varepsilon}\right)\right)$

Each path: supported on $O\left(\text{polylog}\left(\frac{N}{\varepsilon}\right)\right)$ sites

Runtime (# gates): $T = O\left(N\,\text{polylog}\left(\frac{N}{\varepsilon}\right)\right)$

Runtime

Adiabatic runtime: N paths $\times O\left(\log^{1+\alpha}\left(\frac{N}{\varepsilon}\right)\right)$

Each path: supported on $O\left(\text{polylog}\left(\frac{N}{\varepsilon}\right)\right)$ sites

Runtime (# gates): $T = O\left(N\,\text{polylog}\left(\frac{N}{\varepsilon}\right)\right)$

Parallelisation

Circuit depth: $D = O\left(\text{polylog}\left(\frac{N}{\varepsilon}\right)\right)$

RUNTIME

Runtime

Adiabatic runtime: N paths $\times O\left(\log^{1+\alpha}\left(\frac{N}{\varepsilon}\right)\right)$

Each path: supported on $O\left(\text{polylog}\left(\frac{N}{\varepsilon}\right)\right)$ sites

Runtime (# gates): $T = O\left(N\,\text{polylog}\left(\frac{N}{\varepsilon}\right)\right)$

Parallelisation

Circuit depth: $D = O\left(\text{polylog}\left(\frac{N}{\varepsilon}\right)\right)$

Assumption

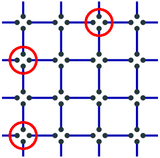
$\Omega(1)$ gap along all N paths

UNIFORM GAP ASSUMPTION

G_{n-1}

↓ change single Q_λ

G_n



$\bigcirc \;=\; Q_\lambda : (\mathbb{C}^D)^{\otimes 4} \to \mathbb{C}^d$

Assumption

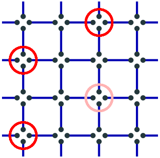
Gap $G_n(s) \geq \text{const} > 0$
$\forall n = 1 \ldots N$ and $s \in [0, 1]$

UNIFORM GAP ASSUMPTION

G_{n-1}

↓ change single Q_λ

G_n



◯ $= \; Q_\lambda : (\mathbb{C}^D)^{\otimes 4} \to \mathbb{C}^d$

Assumption

Gap $G_n(s) \geq \text{const} > 0$

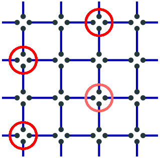
$\forall n = 1 \ldots N$ and $s \in [0, 1]$

UNIFORM GAP ASSUMPTION

G_{n-1}

change single Q_λ

G_n



○ $= \; Q_\lambda : (\mathbb{C}^D)^{\otimes 4} \to \mathbb{C}^d$

Assumption

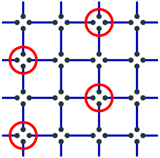
Gap $G_n(s) \geq \text{const} > 0$
$\forall n = 1 \ldots N$ and $s \in [0, 1]$

UNIFORM GAP ASSUMPTION

G_{n-1}

↓ change single Q_λ

G_n



$\bigcirc \;=\; Q_\lambda : (\mathbb{C}^D)^{\otimes 4} \to \mathbb{C}^d$

Assumption

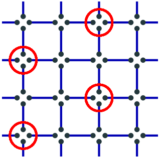
Gap $G_n(s) \geq \text{const} > 0$
$\forall n = 1 \ldots N$ and $s \in [0, 1]$

UNIFORM GAP ASSUMPTION

G_{n-1}

↓ change single Q_λ

G_n



$\bigcirc \;=\; Q_\lambda : (\mathbb{C}^D)^{\otimes 4} \to \mathbb{C}^d$

Assumption

Gap $G_n(s) \geq \text{const} > 0$
$\forall n = 1 \dots N$ and $s \in [0,1]$

Theorem

If $Q_\lambda > q\mathbb{1}$ for all λ and some $q = \text{const} > 0$

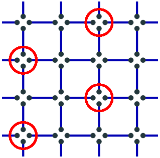
$$\Rightarrow \quad \text{Gap}\, G_n(s) \geq q^{-2}\, \text{Gap}\, G_n(0) \qquad \forall s \in [0,1].$$

UNIFORM GAP ASSUMPTION

G_{n-1}

↓ change single Q_λ

G_n



◯ $= \; Q_\lambda : (\mathbb{C}^D)^{\otimes 4} \to \mathbb{C}^d$

Assumption

Gap $G_n(s) \geq$ const > 0

$\forall n = 1 \dots N$ and ~~$s \in [0,1]$~~ $s = 0$

Theorem

If $Q_\lambda > q\mathbb{1}$ for all λ and some $q =$ const > 0

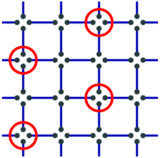
$$\Rightarrow \quad \text{Gap}\, G_n(s) \geq q^{-2}\, \text{Gap}\, G_n(0) \qquad \forall s \in [0,1].$$

UNIFORM GAP ASSUMPTION

G_{n-1}

↓ change single Q_λ

G_n



○ $= \; Q_\lambda : (\mathbb{C}^D)^{\otimes 4} \to \mathbb{C}^d$

Assumption

Gap $G_n(s) \geq \text{const} > 0$

$\forall n = 1 \ldots N$ and ~~$s \in [0,1]$~~ $s = 0$

Theorem

If $Q_\lambda > q\mathbb{1}$ for all λ and some $q = \text{const} > 0$

$$\Rightarrow \quad \text{Gap}\, G_n(s) \geq q^{-2}\, \text{Gap}\, G_n(0) \qquad \forall s \in [0,1].$$

Uniform gap assumption

$$\text{Gap}\, G_n(0) = \Omega(1) \qquad \forall n = 1 \ldots N$$

"Parent Hamiltonian of various system sizes (& shapes) gapped"

COMPARISON WITH OTHER ALGORITHMS

	Runtime (uniform gap)
Our algorithm	$O(N \operatorname{polylog} N)$

COMPARISON WITH OTHER ALGORITHMS

	Runtime (uniform gap)
Our algorithm	$O(N \operatorname{polylog} N)$
	Injective PEPS
[Schwarz et al '12]	$O(N^4)$

COMPARISON WITH OTHER ALGORITHMS

	Runtime (uniform gap)
Our algorithm	$O(N \operatorname{polylog} N)$
	Injective PEPS
[Schwarz et al '12]	$O(N^4)$
	Gibbs states

COMPARISON WITH OTHER ALGORITHMS

	Runtime (uniform gap)	
Our algorithm	$O(N \operatorname{polylog} N)$	
Injective PEPS		
[Schwarz et al '12]	$O(N^4)$	
Gibbs states		
Classical (sampling)		
MCMC (e.g. Metropolis)	$O(N^2)$	
Additional assumptions	$O(N \log N)$	$\leftarrow$ "rapid mixing"

COMPARISON WITH OTHER ALGORITHMS

	Runtime (uniform gap)	
Our algorithm	$O(N \text{ polylog } N)$	
Injective PEPS		
[Schwarz et al '12]	$O(N^4)$	
Gibbs states		
Classical (sampling)		
MCMC (e.g. Metropolis)	$O(N^2)$	
Additional assumptions	$O(N \log N)$	← "rapid mixing"
Quantum		
QSA [Somma et al '08] [Wocjan, Abeyesinghe '08]	$O(\text{poly } N)$	
QM [Temme et al '11]		
QQM [Yung, Aspuru-Guzik '12]		
...		

COMPARISON WITH OTHER ALGORITHMS

	Runtime (uniform gap)		Gap dependence
Our algorithm	$O(N \operatorname{polylog} N)$		$\tilde{O}(\Delta^{-3})$ or $\tilde{O}(\Delta^{-3-6\,\mathrm{dim}})$
Injective PEPS			
[Schwarz et al '12]	$O(N^4)$		$O(\Delta^{-1})$
Gibbs states			
Classical (sampling)			
MCMC (e.g. Metropolis)	$O(N^2)$		$O(\Delta^{-1})$
Additional assumptions	$O(N \log N)$	$\leftarrow$	"rapid mixing"
Quantum			
QSA [Somma et al '08] [Wocjan, Abeyesinghe '08]	$O(\operatorname{poly} N)$		$O(\Delta^{-1/2})$
QM [Temme et al '11]			$O(\Delta^{-1})$
QQM [Yung, Aspuru-Guzik '12]			$O(\Delta^{-1/2})$
...			

Algorithm

- Injective PEPS
- Gibbs distribution

Algorithm

- Injective PEPS
- Gibbs distribution

Ingredients

- Improved adiabatic theorem
- Sequence of local changes
- Lieb-Robinson localisation

Algorithm

- Injective PEPS
- Gibbs distribution

Ingredients

- Improved adiabatic theorem
- Sequence of local changes
- Lieb-Robinson localisation

Algorithm

- Injective PEPS
- Gibbs distribution

Ingredients

- Improved adiabatic theorem
- Sequence of local changes
- Lieb-Robinson localisation

Performance

- Runtime $O(N \operatorname{polylog} N)$ $\leftarrow$ almost optimal
- Depth $O(\operatorname{polylog} N)$
- Almost quadratically faster than classical MCMC algorithms
- Uniform gap assumption

Algorithm

- Injective PEPS
- Gibbs distribution

Ingredients

- Improved adiabatic theorem
- Sequence of local changes
- Lieb-Robinson localisation

Performance

- Runtime $O(N \operatorname{polylog} N)$ $\leftarrow$ almost optimal
- Depth $O(\operatorname{polylog} N)$
- Almost quadratically faster than classical MCMC algorithms
- Uniform gap assumption