# Discriminating N-qudit states using geometric structure

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In the quantum communication, a sender encodes an information through a quantum system and sends it to a receiver who may measure it and extract the information[1-3]. Unlike classical states, nonorthogonal quantum states cannot be perfectly discriminated by performing single measurement. Therefore, in the quantum communication, an optimized procedure is inevitable to discriminate quantum states. The optimization strategy is decided by the result of measurement. The result of measurement can be classified by the certainty in a decision of the given state by a measurement. For instance, if a receiver cannot say about the given state, the measurement is called to be a failure. If quantum states are linearly independent, by permitting the failure of measurement, one can make the confidence of guessing being one, which can eliminate the error of guessing. The strategy is called the unambiguous discrimination[4-7]. If quantum states are linearly dependent, the maximum confidence of guessing cannot be one. However, one can take a strategy of maximum confidence to each guessing, which is called maximum-confidence discrimination[8]. Meanwhile, one can use a minimum-error discrimination(MD)[9], where the failure of measurement is not allowed.

In MD, there may an error in case of discrimination of non-orthogonal quantum states. One wants to minimize the error of guessing quantum states. The general solution of MD to two quantum states was provided by Helstrom[9,10], however, the general solution of MD to three and four quantum states is known only in case of qubit[13,14]. It should be emphasized that the solution is not limited to pure quantum states[11,12] but can be applied to mixed qubit states with arbitrary prior probabilities. Furthermore the general solution of MD to three and four quantum states can lead the strategy to discriminate N-mixed qubit states with arbitrary prior probabilities. In fact, minimum-error measurement of N-qubit states can be always consist of positive-operator valued measure(POVM) with less than five nonzero elements[11,15,16].

Many studies have focused on investigation on optimal measurement when specific conditions are given to quantum states as well as prior probabilities. For example, the square-root measurement[20] is optimal to symmetric pure quantum states with the same prior probability. As known, there exists the necessary and sufficient condition for optimal measurement of minimum error, however it is difficult to find optimal measurement in an analytic form[1,22-24]. Deconinck et al[16] and Bae et al.[26] considered MD of qubit states in view of semidefinite programming[26]. In [13,14] authors provide the necessary and sufficient condition to optimal POVM with nonzero elements by a geometric representation of qubit state, which tells the minimum number of qubit states for minimum error discrimination. We consider the dual one of our problem through the semidefinite programming. For an approach of complementarity problem, we provide not only the condition to the primal and dual problem but also the complementary slackness condition. We use a geometric representation of qudit state to represent the optimality conditions in a geometric way. Through the positive semi-definiteness of POVM, we can understand the element of POVM in terms of geometric representation of qudit state. Using these setting, we can obtain the condition in a geometric form that the non-zero measurement operator should satisfy when the guessing probability is greater than the largest prior probability. We check the condition, by an example that a pretty good measurement can provide minimum error.

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