Extended abstract
Quantum computing with the quad-rail lattice

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Measurement-based quantum computation (MBQC) is a platform for universal quantum computation that only requires the ability to do local adaptive measurements on a pre-prepared highly entangled resource state [1], called a *cluster state* [2]. It shifts part of the difficulty of implementing a quantum computation onto an offline preparation phase, removing the need for controlled entangling operations from the experimental tool box. However, the scalable generation of highly entangled quantum states is still a significant experimental challenge for many candidate physical architectures. Within the optical setting, methods that make use of continuous-variables (CVs) have shown great potential in this regard. Recent theoretical and experimental work has shown that schemes for generating *continuous-variable cluster states* (CVCSs) endowed with specific structural properties—in particular, a bipartite self-inverse graph—can be created from pulses of squeezed light using beamsplitters and delay loops in a highly scalable way [3, 4]. These cluster states are made up of temporally encoded modes, but equivalent states have also been produced in a highly scalable fashion in the frequency domain inside an optical frequency comb [5–7]. In these optical experiments, the limiting resource is the amount of squeezing that can be obtained using nonlinear crystals in the optical cavity. The squeezing strength determines the available entanglement in the resulting cluster state [8]. The imperfect nature of the resource state leads to the accumulation of noise during computation. This finite squeezing effect ultimately limits the length of computation possible, destroying the quantum information even when states are created on perfect laboratory equipment [9–11]. Despite this noise posing a major obstacle for continuous-variable quantum computation, a fault-tolerant scheme exists to combat it by using active error correction [12], and which is amenable to measurement-based protocols using CVCSs.

The raw output resource states produced from the above mentioned scalable schemes inherit a non-standard structure from their construction method, which involves two-mode squeezing and simple interferometry. We call the output state the *quad-rail lattice*. It is shown in Fig. 1 (a). By an additional processing step that involves making some single-mode measurements, the quad-rail lattice can be converted to a 2D square lattice CVCS (shown in Fig. 1 (b)) on which known measurement-based protocols can be applied directly. However, this process "severs" much of the entanglement structure of the quad-rail lattice, thus wasting some of the available experimental squeezing, which directly results in noisier cluster computations [14]. In this work, we present a new measurement protocol that is tailored to run on the quad-rail lattice resource itself, offering two primary advantages over the 2D square lattice method. First, it allows for a theoretically simple

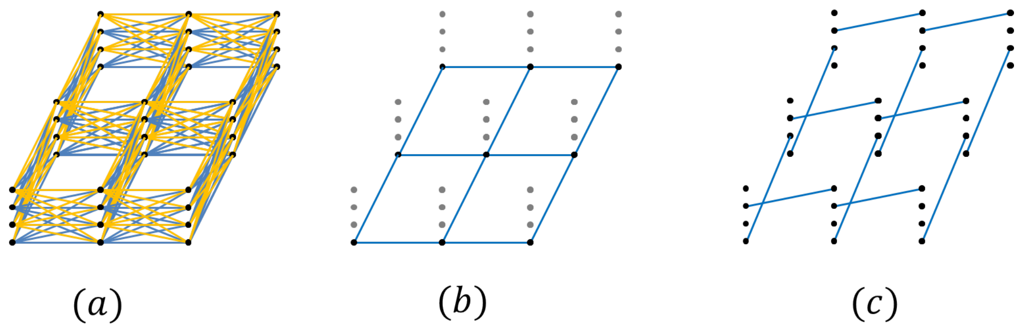

FIG. 1: (**a**) Simplified graphical representation [3, 13] of the quad-rail lattice. On all subfigures, the nodes represent qumodes and the edges represent entanglement between them. In these graphs, blue/yellow edges label the sign of +/− of the edge weights respectively. Structurally, the quad-rail lattice graph is a connected four-layered square lattice. Each lattice site, which we refer to as a *macronode*, comprises of four nodes. (**b**) The quad-rail lattice can be reduced to a continuous-variable cluster state with simpler square lattice graph (shown) by single-mode measurements on all modes in the top three layers. (**c**) By applying a change in the tensor product structure at each macronode, the quad-lattice graph becomes a disjoint collection of linked pairs of nodes. In this picture, local measurements of the physical modes are represented by non-local measurements that can "stitch" the resource structure together for the purposes of gate implementation, as in Fig. 2 (a).

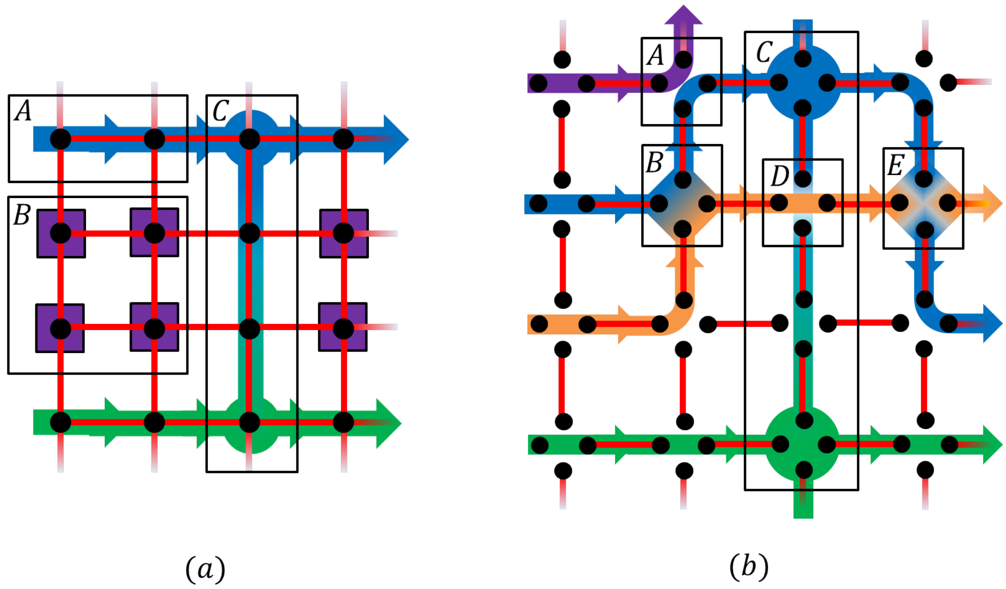

FIG. 2: (**a**) Here we summarize measurement-based quantum computation on the 2D square lattice cluster state. Wires are used for single-mode gates and are embedded into the lattice structure, as shown in (A). They must be separated by "sacrificial" rows of modes, which are measured in the position quadrature basis, denoted by the purple squares, as in (B). Multi-mode gates are implemented by "breaks" in these sacrificial rows that preserve the entanglement structure in the transverse direction.
(**b**) Similarly, wires are used for single-mode gates by our new protocol. However, they can be "threaded" through the cluster in many different ways, and do not require separation by sacrificial qumodes as in the 2D square lattice case. Shown here is an example circuit that can be implemented on the quad-rail lattice (shown using the representation from Fig. 1 (c)). Each inset showcases some "possible moves" that can occur between wires, which propagate logical states via local measurements of the physical modes. In (A) and (B), two wires can connect to the same macronode at right angles, share it without interacting (A) or with the application of a 2-qumode gate (B) depending only on the choice of macronode measurement, and then leave from adjacent ports. In (C), another type of entangling operation, shown between the blue and green wires. This type of gate is similar to how entangling operations are implemented on the regular 2D square lattice CVCS, but with the added advantage that the two connecting wires need not be neighbours. Since this gate preserves the entanglement structure along the horizontal direction, wires can pass over the gate section (such as the orange wire shown above). In (D) and (E), two wires can enter the same macronode and pass through each other without interacting (D) or with and a 2-qumode entangling operation (E).

and experimentally convenient method for implementing multi-mode gates. Second, by preserving the QRL structure, our new protocol uses squeezing resources more efficiently, thereby extending the length of possible computation.

A distinguishing feature of our protocol is that it takes advantage of the macronode structure of the quad-rail lattice. Here a macronode is a multi-mode lattice site comprised of four physical modes (see Fig. 1 (a)). Each logical mode is encoded within a macronode, rather than an individual cluster node, as is usually the case for MBQC. Moreover, macronodes can contain up to two logical modes. The logical input states are encoded with respect to an alternative tensor product decomposition to the one described by the physical modes, as shown in Fig. 1 (c). We characterize the effect of physical mode measurements on the logical modes, providing a "measurement blueprint" for gate implementation. Each macronode offers more measurement degrees of freedom than regular cluster nodes, effectively providing more "handles" for transforming the encoded input state. Note that this approach comes at no additional experimetal cost over MBQC using the 2D square lattice state produced from the quad-rail lattice. These advantages are expressed via example in Fig. 2 and by comparison to a 2D square cluster state computation on a lattice with the same size.

This work provides a theoretically simple framework for MBQC on the quad-rail lattice that takes advantage of its macronode structure. It improves on previous work by having improved noise properties and offering a convenient and more compact method for implenting gates via measurements. Both of these advantages are expected to benefit fault-tolerant implementations of a measurement-based quantum computer that uses continuous-variables.

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