

The minimal form of ancilla mediated quantum computation

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Since the potential power of a quantum computer was first realised there have been huge developments in the theory of the subject and many proof-of-principle experiments utilising a large range of candidate physical systems. Although these experiments demonstrate an impressive, and improving, control over quantum systems they are a long way from the realisation of a quantum computer that can outperform the existing classical devices. Hence, there is great interest in developing theoretical models for quantum computation that take into consideration the difficulties in engineering quantum systems and are therefore more directly applicable to experimental realisations of a quantum computer.

The original theoretical setting for quantum computation is the gate (or circuit) model, however, although theoretically elegant, the physical requirements of a direct implementation of this model are not easy to realise. Two-qubit interactions between arbitrary pairs of qubits must be facilitated alongside the competing requirement that individual qubits can be addressed and isolated, in order to implement single qubit gates and measurements and preserve the quantum information. Indeed, most experimentally implemented or proposed schemes mediate the required multi-qubit interactions using some ancillary system, with the original Cirac-Zoller ion-trap gate [4] a well known example. We refer to schemes in which all multi-qubit interactions are mediated via some ancillary system as *ancilla-mediated* models [10, 11] and the concept behind these models is shown schematically in Fig. 1.

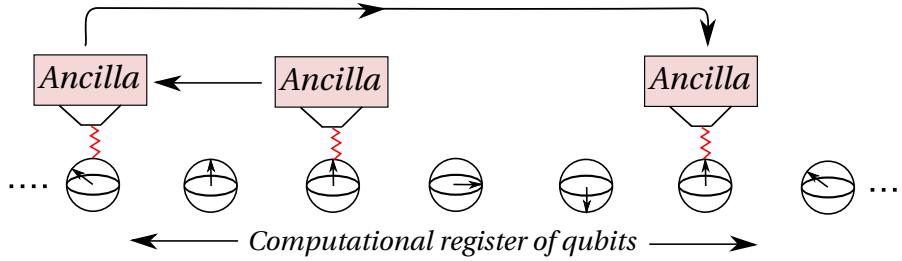


Figure 1: A schematic representation of ancilla-mediated quantum computation. Register qubits that are well isolated from each other, and may be optimised for long coherence times, interact in turn with some ancillary quantum system that mediates interactions between these qubits. The induced interactions between the register qubits allow universal quantum computation to be implemented on the register. Here the ancilla is represented as moving between static qubits, however the ancilla need not be of this form - for example it could be a shared vibrational mode of the qubits.

Although mediating the register-qubit interactions via an ancilla enables better isolation of the register qubits, which is essential for long coherence times, these models may still require multiple forms of precise control over the system as a whole. For example, the well studied ancilla-mediated model known

as *quantum bus computation* [14, 8] requires multiple forms of ancilla-register interaction¹ [10] and local operations on individual register qubits. It is interesting from both a practical and a fundamental point of view to consider what is the minimal required control over an ancilla and register composite system whilst still being able to implement universal quantum computation on the register. Some control over and access to the register is unavoidable so we consider imposing the constraint that

The only access to the register of isolated qubits is via a single fixed-time interaction between a single register qubit at a time and the ancillary system.

This constraint implies that not only are the two-qubit gates implemented via an ancilla but no other operations, such as single qubit gates or measurements, may be implemented directly on the register. Furthermore the interactions with the ancilla are limited to being of a single form - hence only a single fixed interaction needs to be physically characterised and controlled and the setup may be optimised to achieve this. A further challenging problem in any implementation is reliable single-qubit measurements and hence it is desirable to keep these to a minimum by requiring that:²

Measurements of the system are only required at the end of the computation.

Finally, although we have restricted the manipulations of the register to a minimum, control over the ancilla may also be physically challenging and hence we further demand that:

Manipulation of the ancilla is restricted to ancilla preparation.

We may further restrict this to ancilla initialisation in easily prepared states, in the case of a qubit we take this to be the two computational basis states, denoted $|0\rangle$ and $|1\rangle$ respectively. This means that no additional control of the ancilla is hidden in the state-preparation step.

The three constraints presented above, when taken together, would hugely simplify the practical requirements of a model - it is only necessary to engineer one fixed interaction and prepare the ancilla in two orthogonal states. However, it is not immediately obvious that universal quantum computation can be achieved in a such a simple and restricted fashion and hence it is also of fundamental interest to consider whether this is possible. We now present a non-technical description of a model of computation that fulfils all of these physically motivated constraints and hence requires the minimal level of control of the composite ancillary and register system. This model employs an ancilla qubit and the full details of this work can be found in Ref. [11] building on our previous work in Ref. [9]. We describe how we may implement universal quantum computation on a register of qubits by showing how we may implement a two-qubit entangling gate and a universal set for single qubit gates on arbitrary register qubits. It is well known that such a set is sufficient for universal quantum computation [2].

We first describe how we may implement an entangling gate between two register qubits. It is not possible to mediate an entangling gate between two qubits via sequential interactions with an ancilla when restricted to globally unitary evolution [7] and hence the minimum possible number of ancilla-register interactions to implement the essential entangling gates on the register is three - the number required in our model. In order to implement an entangling gate on a pair of qubits the ancilla, prepared in the fixed initial state $|0\rangle$, interacts with the qubits sequentially before then interacting for a second time with the first qubit. This is represented schematically in Fig. 2a. The form of the entangling gate induced on the register depends on the specific form of the ancilla-register interaction. The possible forms for the ancilla-register interaction are a subset of those that are locally equivalent to $\text{SWAP} \cdot C(R(\theta))$ where

¹This can be replaced by local control of the ancilla in the form of the appropriate Fourier transform.

²This constraint is clearly not met by the measurement-based ancilla-driven models of Ref. [1, 5, 6]. For simplicity discussion of these interesting models and their relation to this work has not been included but can be found in the full-length version of this work [11] and our proceeding work of Ref. [9]

SWAP is the well known gate that exchanges the states of the incident qubits and $C(R(\theta))$ is a controlled rotation by θ - it applies a phase of $e^{i\theta}$ if both qubits are in the $|1\rangle$ computational basis state.

It is now only necessary to describe how a universal set of single qubit unitaries may be implemented on each register qubit. This can be done by interacting a register and ancilla qubit twice - the gate implemented on the register depends on which of two computational basis states the ancilla qubit is initialised too. This is shown schematically in Fig. 2b. The single qubit gate u_i is implemented by initialising the ancilla to the computational basis states $|i\rangle$, $i = 0, 1$. The precise form of u_0 and u_1 is dependent on the choice of interaction and hence for this model to be universal the form of the ancilla-register interaction is restricted to those for which the implemented u_0 and u_1 form a universal set for single qubit gates. An example of a possible ancilla-register interaction is of the form

$$\text{Interaction} = \mathbb{I} \otimes H \cdot \text{SWAP} \cdot C(R(\pi/4)), \quad (1)$$

where the first (second) term in the tensor product is the register (ancilla) qubit, \mathbb{I} is the identity operator and H is the well known Hadamard gate. In this case the induced entangling gate on a pair of register qubits is locally equivalent to $\text{SWAP} \cdot C(R(\pi/4))$ and can simulate CNOT in a straightforward manner. Furthermore, the single qubit set that can be implemented on the register is $u_0 = H$ and $u_1 = R(\pi/4)HR(\pi/4)$. We have presented a proof of the universality of this set in the Ref. [11].

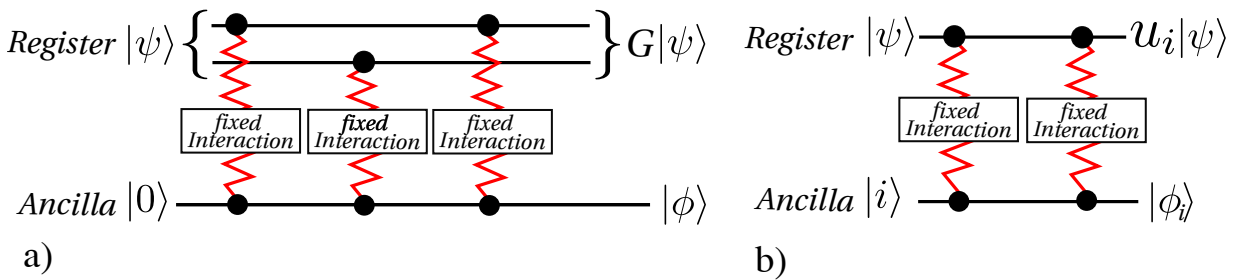


Figure 2: A schematic representation of how a) the two qubit entangling gate, denoted G , and b) the single qubit gate u_i with $i = 0, 1$ are implemented in the minimal model presented herein. The forms the interaction can take are discussed in the text and the forms of G , u_0 and u_1 depend on this.

The model presented herein requires minimal control over both the register and ancillary systems as it obeys the constraints highlighted in the text. The only requirement for the implementation of this model is the characterisation of a single appropriate fixed-time interaction between an ancillary and register qubit. The physical systems used may be optimised for long coherence times in the case of the register qubits and both interaction strength and communication for the ancilla qubits - a highly relevant example being given by register qubits realised in spin ensembles in diamond coupled via ancillary superconducting flux qubits [12]. This scheme is well suited to a variety of physical realisations with further interesting examples given by the coupling of nuclear spins via ancillary electronic spins in nitrogen-vacancy defects [15] or the coupling of atomic [13] or spin [3] qubits via ancillary photons. A more detailed discussion of possible realisations and appropriate physically reasonable Hamiltonians is given in Ref. [11]. This model brings the abstract theory of quantum computation closer to the physical reality of what is experimentally achievable, giving a possible method for realising quantum computation with a practical and simple scheme. Furthermore it sheds light on the minimal resources required for universal quantum computation.

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