Quantum entanglement can be simulated without communication

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(joint work with Nicolas Gisin, Serge Massar, and Sandu Popescu)

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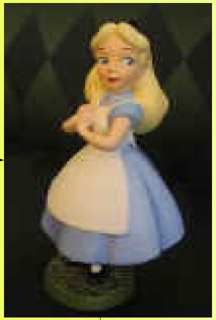
Simulation of E.P.R. experiment

$| \Psi^{-} >$

Input: $\vec{a}$

Input: $\vec{b}$

Alice



Bob

Output: $A \in \{-1, +1\}$

Output: $B \in \{-1, +1\}$

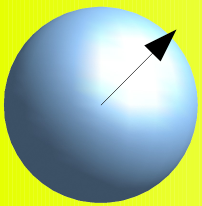
$$P(A, B) = \frac{(1 - AB\ \vec{a} \cdot \vec{b})}{4} \Leftrightarrow E(AB) = -\vec{a} \cdot \vec{b}$$

$P(A) = 1/2\ \forall \vec{b}$ ← CAUSALITY → $P(B) = 1/2\ \forall \vec{a}$

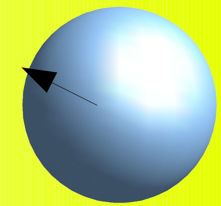
Local Hidden Variable (LHV) Model

Shared randomness: λ

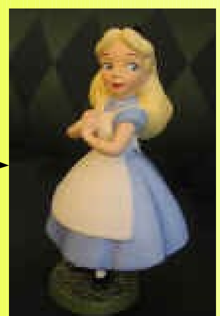
Input: $\vec{a}$



Input: $\vec{b}$



Alice



Bob

Output: $A(\vec{a},\lambda) \in \{-1,+1\}$

Output: $B(\vec{b},\lambda) \in \{-1,+1\}$

$$E(AB|\vec{a},\vec{b}) = \int_{\lambda\in\Lambda} p(\lambda)\; A(\vec{a},\lambda)\; B(\vec{b},\lambda)$$

$$\stackrel{?}{=} -\vec{a}\cdot\vec{b}$$

BUT...

Bell's Theorem:

No Local Hidden Variable model can simulate the quantum correlations of the EPR experiment

Indeed, any LHV model must satisfy the CHSH inequality:

$$|C(\vec{a}_0,\vec{a}_1,\vec{b}_0,\vec{b}_1)|\leq 2 \quad \forall\, \vec{a}_0,\vec{a}_1,\vec{b}_0,\vec{b}_1 \in S_2$$

with $C(\vec{a}_0,\vec{a}_1,\vec{b}_0,\vec{b}_1)=E(AB|\vec{a}_0,\vec{b}_0)+E(AB|\vec{a}_0,\vec{b}_1)+E(AB|\vec{a}_1,\vec{b}_0)-E(AB|\vec{a}_1,\vec{b}_1)$

In quantum mechanics:

$$\exists\ \vec{a}_0,\vec{a}_1,\vec{b}_0,\vec{b}_1 \in S_2 \quad \textit{such that} \quad C(\vec{a}_0,\vec{a}_1,\vec{b}_0,\vec{b}_1)=2\sqrt{2}$$

- So we need extra resources, in addition to those allowed by any Local Hidden Variable model
- The amount of extra resources that is needed gives us some measure of the non-locality of QM (Maudlin 92; Brassard, Cleve, Tapp 99)

Additional resources

Classical communication : in number of bits (on average or in worst case)

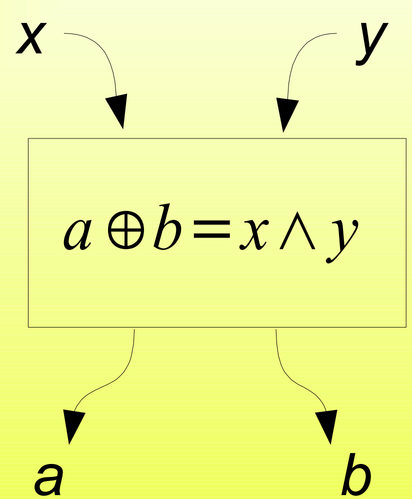
→ Allows for superluminal communication

Freedom to post-select (detection loophole) : the parties are given the possibility to output “no result”, simulating an imperfect detector

→ Does not allow for superluminal communication but probabilistic

Non-Local Box : in number of uses

→ Remains causal : strictly weaker resource than 1 bit of communication



Popescu and Rohrlich 94
van Dam 00

$$x, y, a, b \in \{0,1\}$$

Outline of the known protocols

Resource	**Amount**	$\vec{a}, \vec{b}$	**Reference**
Communication	1.17 bit on Average	Equator	Maudlin 92
Communication	8 bits in Worst Case	Sphere	Brassard, Cleve, Tapp 99
Communication	1.48 bit on Average	Equator	Steiner 99
Post-Selection	P(A_output)= P(B_output)= 2/3	Sphere	Gisin, Gisin 99
Communication	1.19 bit on Average	Sphere	NJC, Gisin, Massar 00
Communication	1 bit in Worst Case	Sphere	Toner, Bacon 03
Non-Local Box	1 use in Worst Case but no communication	Sphere	**(this talk)**

Non-Local Box

- Maximally non-local : maximally violates CHSH inequality $C=4$
- Causal

+1	+1	+1	−1

$$C(\vec{a}_0,\vec{a}_1,\vec{b}_0,\vec{b}_1)=E(AB|\vec{a}_0,\vec{b}_0)+E(AB|\vec{a}_0,\vec{b}_1)+E(AB|\vec{a}_1,\vec{b}_0)-E(AB|\vec{a}_1,\vec{b}_1)$$

x=0 y=0 x=1 y=1

a and **b** are anticorrelated when **x** = 1 and **y** = 1, otherwise they are correlated

x y

$$x\wedge y=a\oplus b$$

$$A=1-2a \qquad B=1-2b$$

$$\{0,1\}\rightarrow\{+1,-1\}$$

$$x,y,a,b\in\{0,1\}$$

$$p(a=0|x,y)=p(a=0|x)=\frac{1}{2}$$

$$p(b=0|x,y)=p(b=0|y)=\frac{1}{2}$$

Is it a <u>sufficient</u> resource to simulate <u>any VN measurement</u> on an EPR state?

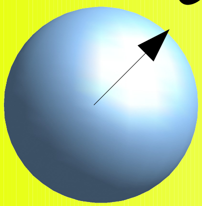
- It is sufficiently nonlocal (more than QM !)
- It is causal (just like QM !) : does not “spoil” resources
- It admits binary inputs, while there are infinitely many possible VN measurements

HOW DOES IT WORK ? **Next slide**

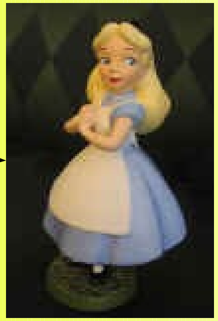
WHY DOES IT WORK ? **Next talk**

$$\vec{\lambda}_1, \vec{\lambda}_2$$

$\vec{a}$

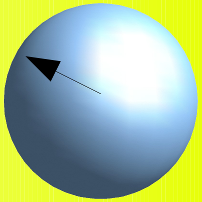


Alice



Bob

$\vec{b}$



$$x = sgn(\vec{a}\cdot\vec{\lambda}_1) + sgn(\vec{a}\cdot\vec{\lambda}_2)$$

with $$sgn(t) = 0 \quad t > 0$$
$$\quad\quad = 1 \quad t \leq 0$$

$$y = sgn(\vec{b}\cdot\vec{\lambda}_+) + sgn(\vec{b}\cdot\vec{\lambda}_-)$$

with $$\vec{\lambda}_\pm = \vec{\lambda}_1 \pm \vec{\lambda}_2$$

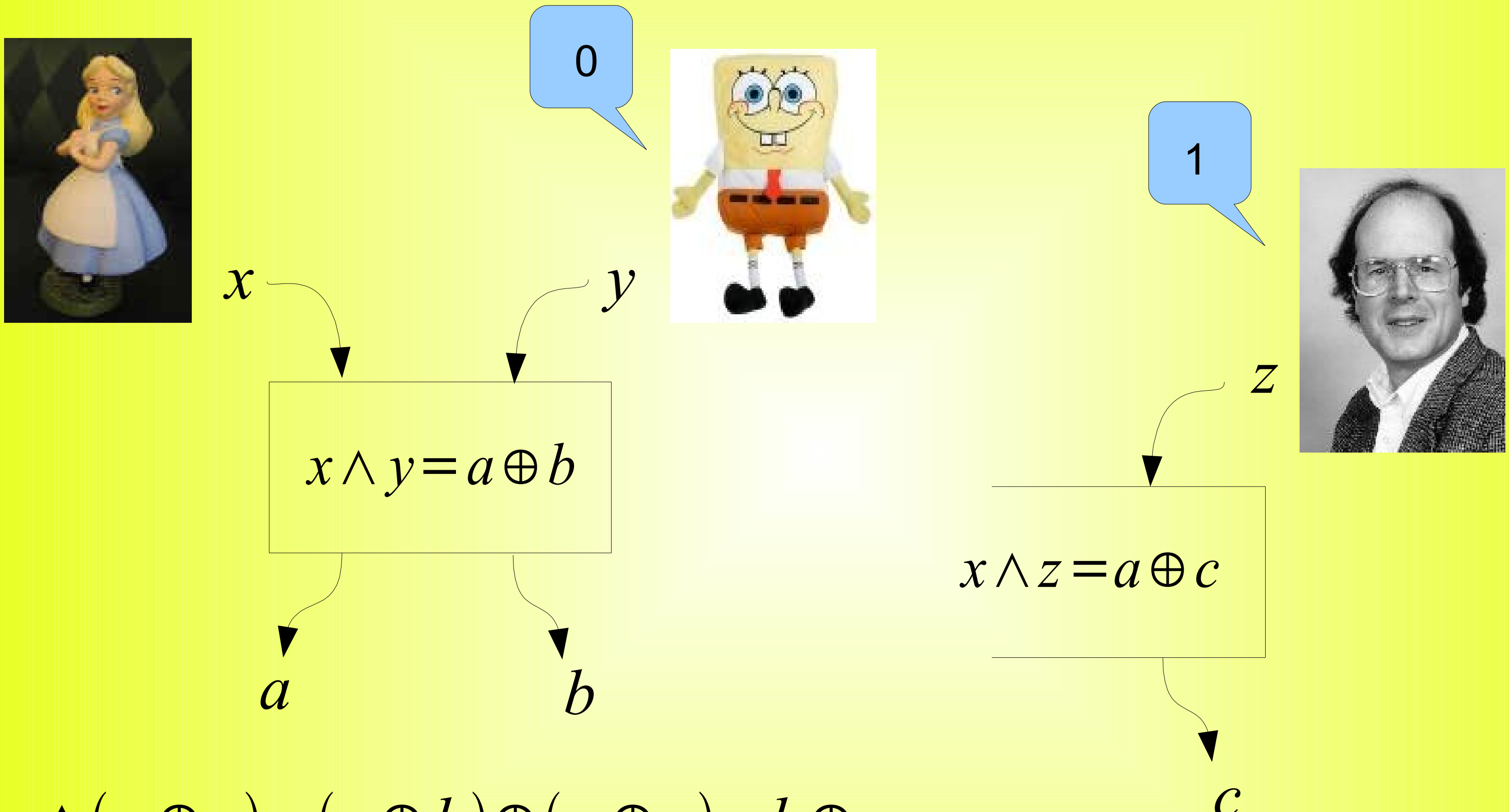
$$x \wedge y = a \oplus b$$

$$A(\vec{a}, \vec{\lambda}_1, \vec{\lambda}_2) = 1 - 2[a + sgn(\vec{a}\cdot\vec{\lambda}_1)]$$

$$B(\vec{b}, \vec{\lambda}_1, \vec{\lambda}_2) = -1 + 2[b + sgn(\vec{b}\cdot\vec{\lambda}_+)]$$

RESULT: $$E(AB) = -\vec{a}\cdot\vec{b}$$

Monogamy : Non-Local Box cannot be shared



$$x \wedge (y \oplus z) = (a \oplus b) \oplus (a \oplus c) = b \oplus c$$

$$y = 0 \ \wedge \ z = 1 \longrightarrow b \oplus c = x$$ **Non causal !**

- Exploit monogamy to do QKD (talk by N. Gisin, A. Acin, L. Masanes)
- Characterize monogamy in general (talk by B. Toner)

Conclusion & Perspectives

- Extend to non-maximally entangled states

  1 use of Non-Local Box is not sufficient
  N. Brunner, N. Gisin, V. Scarani, 05

  Non-maximally entangled state is "more non-local"

- Extend to POVM measurements (related)

- Extend to multipartite states and/or higher dimensions

Non-Local Box *appears to be useful conceptual tool (non-locality characterization, secret key distribution, communication complexity, bit commitment,...)*